UT-1/J-25: 51014
CALCULUS AND LINEAR ALGEBRA
BS-133

M.M. 75
Time-3Hrs

Note: All questions in Part-A and Part-B are compulsory. Attempt any four questions from Part-C selecting at least one from each unit.

PART-A (15 Marks)

Q. No. – 1 Answer the following questions. 15x1=15

(i) Define Rolle's theorem.

(ii) Find $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

(iii) Find $\lim_{x\to 0}(\frac{logx}{cotx})$.

(iv) A set $\{v\}; v \neq 0$ is LD or LI?

(v) If $A=\begin{bmatrix}0 & 1\\ 2 & -1\end{bmatrix}=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$ Where $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then A is…?

(vi) Any set of vectors which includes the zero vector is always LI or LD?

(vii) Give an example which shows that R is not a vector space over C.

(viii) Give an example which shows that Q is not a vector space over R.

(ix) Determine whether the following mapping is an LT.
$T: R^3 \to R^2$defined by $T(x,y,z)=(x+1,y+z)$

(x) If every minor of order r of a matrix A is zero then the rank of matrix A is…?

(xi) Determine whether the following polynomials are linearly independent or not?
$x^2+x+1, x, 1$

(xii) Give examples of three fields which are vector space over itself.

(xiii) The sum of two eigen values and trace of a 3×3 matrix A are equal, then the value of det(A) is…...?

(xiv) If the eigen values of an orthogonal matrix are all real then the eigen values are always…..?

(xv) Eigen values of a skew symmetric matrix are ….?

PART-B (20 Marks)

UNIT-I

2 Prove that $\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x^2\sin x}=\frac{1}{3}$ 5

UNIT-II

3 Use Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$. 5

UNIT-III

4 Determine whether or not the following set of vectors are LD or LI 5
.$\{(1,-2,5,-3),(0,7,-9,2),(0,0,1,0),(0,0,0,1)\}$

UNIT-IV

5 Which of the following matrices are diagonalizable. 5
$(i)\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ (ii)$\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$.

PART-C (40 Marks)

UNIT-I

6 Prove that $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma m+n}$. 10

7 Find the volume generated by the revolution of the curve $27ay^2=4(x-2a^2)$ about the 10
$x-axisf(x)=x^2+cosx$ on $[-\frac{\pi}{4},\frac{\pi}{4}]$

UNIT-II

8 Reduce the following matrix to normal form and hence find the rank $\begin{bmatrix}1 & 0 & 2 & 1\\ 0 & 1 & -2 & 1\\ 1 & -1 & 4 & 0\\ -2 & 2 & 8 & 0\end{bmatrix}$ 10

9 Investigate for consistency of the following equation and hence find the solution 10
$4x-2y+6z=8, x+y-3z=-1, 15x-3y+9z=21.$

UNIT-III

10 Let $M_{22}(R)$ denote the vector space of all 2×2 matrices over reals. Determine the following is subspace of $M_{22}(R)$ or not 10

$$W=\left\{\begin{bmatrix}a & b\\ c & 0\end{bmatrix}: a,b,c\in R\right\}$$

11 Verify rank and nullity theorem for the linear transformation. 10
$T(\varkappa_1,\varkappa_2,\varkappa_3,\varkappa_4)=(\varkappa_1-\varkappa_2+\varkappa_3+\varkappa_4,\varkappa_1+2\varkappa_3-\varkappa_4,\varkappa_1+\varkappa_2+3\varkappa_3-3\varkappa_4)$

UNIT-IV

12 Find the eigen values and corresponding eigen vectors of the linear operator $T: R^3\to R^3$ defined 10
By: $T(\varkappa,y,z)=(\varkappa-y,2\varkappa+3y+2z,x+y+2z)$

13 Diagonalize the matrix $\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. 10

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