

## BT-4 / M-18

## MATHEMATICS-III

## Paper-AS-201N

Time allowed : 3 hours]

[Maximum marks : 75

Note :- Attempt any five questions, selecting at least one from each Part.

## Part-A

1. (a) Find the Fourier series expansion for  $f(x)$ , if

$$f(x) = -\pi, -\pi < x < 0$$

$$x, 0 < x < \pi$$

7½

Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

- (b) Find the half range sine series for  $f(x) = x \cos x$  in  $(0, \pi)$  7½

2. (a) Find the Fourier cosine transform of  $e^{-ax}$ ,

7½

hence evaluate  $\int_0^{\infty} \frac{\cos \lambda x}{x^2 + \alpha^2} dx$

- (b) Verify convolution theorem for  $f(x) = g(x) = e^{-x^2}$  7½

3. (a) Use De Moivre's theorem to solve the equation 7½

$$x^9 + x^5 - x^4 - 1 = 0$$

- (b) If  $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = A + iB$ , Prove that

(i)  $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \dots (a_n^2 + b_n^2) = A^2 + B^2$ . 4

(2)

$$(ii) \tan^{-1} \frac{b_1}{a_1} + \tan^{-1} \frac{b_2}{a_2} + \dots + \tan^{-1} \frac{b_n}{a_n} = \tan^{-1} \frac{B}{A}$$

3½

### Part-B

4. (a) Find the regular function whose imaginary Part is  $e^{-x}(x \sin y - y \cos y)$  7½
- (b) Find the bilinear transformation which maps the points  $z = 1, i, -1$  in to the points  $w = 0, 1, \infty$ . 7½
5. (a) Solve the PDE. 7½
- $$(y+z)p - (z+x)q = x - y, \text{ where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$
- (b) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$ . 7½
6. Using simplex method, solve 15
- Maximize  $Z = 4x_1 + 3x_2 + 6x_3$
- Subject to  $2x_1 + 3x_2 + 2x_3 \leq 440$ ;
- $4x_1 + 3x_3 \leq 470$ ;  $2x_1 + 5x_2 \leq 430$ ,
- $x_1, x_2, x_3, \geq 0$

### Part-C

7. (a) A purse contains 2 silver and 4 copper coins and a second purse contains 4 silver and 4 copper coins. If a coin is

(3)

selected at random from one of the two purses, what is the probability that it is a silver coin?  $7\frac{1}{2}$

- (b) A factory production line is manufacturing bolts using three machines A, B and C of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machine, that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from machine A?  $7\frac{1}{2}$

8. (a) A function is defined as follows:

$$f(x) = 0, \quad x < 2$$

$$= \frac{1}{18} (2x + 3), \quad 2 \leq x \leq 4$$

$$= 0, \quad x > 4$$

Show that it is a density function. Find the probability that a variate having this density will fall in the interval  $2 \leq x \leq 3$ ?

- (b) Out of 800 families with 5 children each, how many would you expect to have

(a) 3 boys (b) 5 girls (c) either 2 or 3 boys?

Assume equal probabilities for boys and girls.  $7\frac{1}{2}$

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