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BT-3/ D-21: 43135

BS-205 A : Mathematics-III

Time: 3 Hours] **[Max. Marks: 75**

Note: Attempt any five questions.

1 (a) Test the convergence of $\sum_{n=1}^{\infty}\left(\frac{(n+1)(n+2)}{n^2\sqrt{n}}\right)$

(b) Discuss the convergence or divergence of the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3\cdot 4}+\cdots \qquad x>0$$

2 (a) Expand $f(x)=x\sin x$ as a Fourier series in $(0,2\pi)$.

(b) Find the half -range sine series for $f(x)=x(\pi-x)$ in the interval $(0,\pi)$ and deduce that

$$\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+--=\frac{\pi}{32}$$

3 (a) Solve $(x^2y-2xy^2)\,dx-(x^3-3x^2y)\,dy=0$ using exact differential equation.

(b) Solve the differential equation

$$x^2\left(\frac{dy}{dx}\right)^2+xy\frac{dy}{dx}-6y^2=0$$

4 (a) Solve $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=e^{2x}$

(b) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$$

5 (a) Change the order of integration in the interval:

$$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2(ax)^{\frac{1}{2}}} dy\,dx$$

(b) Show that area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$

6 (a) Evaluate

$$\int_1^{e}\int_1^{\log y}\int_1^{e^x} \log z \, dz \, dx \, dy$$

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

7 (a) If $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, show that $\Delta r^n = nr^{n-2}\vec{r}$

(b) Prove that $\nabla^2 f(r) = f''(r)+\frac{2}{r}f'(r)$

8 (a) Evaluate the line integral $\int_C (x^2+xy)\,dx+(x^2+y^2)\,dy$,

where C is the square formed by the lines $x=\pm1, y=\pm1$

(b) Verify Green's Theorem for $\oint_c [(xy+y^2)\,dx+x^2\,dy]$,

where c is bounded by $y=x$ and $y=x^2$.