

Roll No.

Total Pages : 04

BT-3/D-20

43135

MATHEMATICS-III

BS-205A

Time : Three Hours]

[Maximum Marks : 75

Note : All questions in Part A and Part B are compulsory.
Attempt any *four* questions from Part C, selecting *one* question from each Unit.

Part A

1. (a) Determine the following series converges or

diverges $\sum_{n=2}^{\infty} \frac{1}{n^5 - n^2 - 1}$.

(b) Solve $3y' + xy = xy^{-2}$.

- (c) Find the solution of :

$$(D^2 + 4D + 4)y = 5 \cos x.$$

- (d) Evaluate the integral :

$$\int_0^1 \int_0^x \int_0^{1+2x+3y} f(x, y, z) dx dy dz, \text{ where } f(x, y, z) = 5.$$

- (e) Evaluate Curl of $e^{xyz}(\mathbf{i} + \mathbf{j} + \mathbf{k})$ at the point (1, 2, 3).

5×3=15

Part B

2. Determine whether the series converge :

(a) $\sum_{n=1}^{\infty} \frac{n}{e^n}$.

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{5^n}$. 5

3. Solve : 5

$$\left(y + \sqrt{x^2 + y^2} \right) dx - x dy = 0, y(1) = 0.$$

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$. 5

5. Calculate $\nabla^2 f$ when $f = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$ at the point $(1, 1, 0)$. 5

Part C

Unit I

6. (a) Define Cauchy first root test for sequence. Also check the convergence of $\langle a_n \rangle$, where

$$a_n = \left(\frac{n^3 + n}{n + 5} \right)^{\frac{1}{n}}. \quad 5$$

(b) Series $\sum \frac{1}{n!}$ converges or diverges ? Justify. 5

7. Expand $f(x) = x \sin x$ as a Fourier series in $(0, 2\pi)$. 10

Unit II

8. (a) Find the general solution and singular solution of
 $y = \sin(y - xp)$. 5

(b) Solve $y(2x^2 - xy + 1)dx + (x - y)dy = 0$ using exact differential equation. 5

9. (a) Solve : 5

$$(D^2 - 4D - 5)y = e^{2x} + 3 \cos(4x + 3)$$

(b) Solve the following differential equation using method of variation of parameter

$$(D^2 - 2D)y = e^x \sin x. \quad 5$$

Unit III

10. Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}$ over the positive quadrant of the circle $x^2 + y^2 = 1$. 10

11. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$. **10**

Unit IV

12. Show that $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$ for any vector function \bar{A} . **10**
13. Verify the Stokes theorem for $\bar{A} = y^2\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$. **10**