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BT-4/M-21 **44182**

DISCRETE MATHEMATICS

Paper–PC-IT-204A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Show that

$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

by mathematical induction.

(b) Construct the truth tables for the following statements. (any two out of three)

(i) $p \to p$.

(ii) $(p \to p) \vee (p \to \bar{p})$.

(iii) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

2. (a) If A, B, C be arbitrary sets, prove that

(i) $(A \cap B) \subseteq B \subseteq (A \cup B)$.

(ii) $(A \cap B) \subseteq A \subseteq (A \cup B)$.

(b) Out of 200 students, 50 of them take the course Discrete Mathematics, 140 of them take the course Economics, and 24 of them take both courses. Since both courses have scheduled examinations for the following day, only students who are not in either one of these courses will be able to go the party the night before. Find how many students will be at the party ?

UNIT–II

3. (a) Let A = {1, 2, 3} and B = {r, s}, find $A \times B$ and $B \times A$ and verify $A \times B \neq B \times A$.

(b) Define the following :

(i) Reflexive relation.

(ii) Anti-symmetric relation.

(iii) Transitive relation.

4. (a) Let R be the relation from A to B, and let A_1 and A_2 be subsets of A. Prove that

(i) If $A_1 \subseteq A_2$ then $R(A_1) \subseteq R(A_2)$.

(ii) $R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)$.

(b) Let R and S be relations on a set A. Prove that

(i) If R is reflexive, so is R^{-1}.

(ii) If R and S are reflexive, then so are $R \cap S$ and $R \cup S$.

UNIT–III

5. (a) Prove that if $f : A \to B$ and $g : B \to C$ are onto functions, then $g \circ f$ (composition function of g and f) is onto.

(b) Suppose that repetitions are not permitted. How many four-digit numbers can be, formed from the six digits 1, 2, 3, 5, 7, 8 and how many of them are less than 4000?

6. (a) Solve the recurrence relation $a_{r+2} - 2a_{r+1} + a_r = 2^r$ by method of generating functions with initial conditions $a_0 = 2$ and $a_1 = 1$.

(b) State and prove Pigeon hole Principle.

UNIT–IV

7. (a) Define the following :

(i) Semi group.

(ii) Monoid.

(iii) Group.

(b) Consider an algebraic system (G, *), where G is set of all Non-zero real numbers and * is binary operation defined by $a*b = \frac{a.b}{4}$. Show that (G, *) is an Abelian group.

8. (a) Prove that every subgroup of a cyclic group G is Cyclic.

(b) Let R is a ring with unity and $(x.y)^2 = x^2 .y^2 \ \forall x, y \in$ R. Show that R is a commutative ring.