

Roll No. ....

Total Pages : 04

GSE/D-22

1163

MATHEMATICAL FOUNDATIONS-I

BCA-113

Time : Three Hours]

[Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Draw Venn diagram for  $A \cup B$ .  
(b) Define Lattices.  
(c) Evaluate :

$$\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$$

- (d) Find order and degree of the differential equation

$$\frac{d^3 y}{dx^3} - \left(\frac{dy}{dx}\right)^{1/3} = xy.$$

- (e) Solve the differential equation :

$$\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - y = 0.$$



## Unit I

2. (a) Prove that :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- (b) Show that if  $R_1$  and  $R_2$  are equivalence relation on  $A$  then  $R_1 \cap R_2$  is an equivalence relation.

3. (a) A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw ?

- (b) If  $f(x, y, z) = (x \cup y) \wedge (x \cup y') \wedge (x' \cup z)$  be a given Boolean function, determine its DN form.

## Unit II

4. (a) Prove that limit of a function at a point, if exists, is unique.

- (b) Find the value of  $a$  if the function  $f$  given by

$$f(x) = \begin{cases} 2x-1, & 2 < x \\ a, & x = 2 \\ x+1, & x > 2 \end{cases} \text{ is continuous at } x = 2.$$

5. (a) Find  $\frac{dy}{dx}$  if  $y = \frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}}$ .

- (b) Find  $\frac{dy}{dx}$  if  $y = (\sqrt{x})^x + (x)^{\sqrt{x}}$ .



(c) If  $y = x^x$ , show that :

$$\frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0.$$

### Unit III

6. (a) Form the differential equation of the equation  $(x - a)^2 + (y - b)^2 = r^2$  by eliminating the arbitrary constants  $a$  and  $b$ .

(b) Solve the differential equation :

$$y\sqrt{1-x^2}dy + x\sqrt{1-y^2}dx = 0.$$

7. (a) Solve the differential equation :

$$(x^2 - y^2)dx - xydy = 0.$$

(b) Solve the differential equation :

$$(x^2 + y^2 + 2x)dx + 2ydy = 0.$$

### Unit IV

8. (a) Solve the differential equation :

$$\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}.$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x.$$

9. (a) Solve the differential equation :

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$

- (b) Determine the curve in which the length of the sub-normal is proportional to the square of the ordinate.

