Roll No. .... **Total Pages : 04**

BCA/M-20 **1889**

MATHEMATICAL FOUNDATION-II

BCA-123

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. Attempt *four* more questions selecting exactly *one* question from each Unit. All questions carry equal marks.

(Compulsory Question)

1. Explain the following :

(a) Logical statement

(b) Universal quantifer

(c) Binary operation

(d) Isomorphism of a ring

(e) Matrix

(f) Row echelon matrix

(g) Characteristic equation

(h) Hermitian matrix. **8×2=16**

Unit I

2. (a) If p and q be any statements, then construct the truth table of $\sim p \wedge q$ and $(p \rightarrow q) \rightarrow (p \wedge q)$. **8**

(b) By using laws of algebra of logical statements, prove that $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ and $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$. **8**

3. (a) Prove by the principle of mathematical induction that the sum of first n natural numbers $= n(n + 1)/2$ i.e., $1 + 2 + 3 + ... + n = n(n + 1)/2$, for all $n \varepsilon$ N. **8**

(b) Using principle of mathematical induction, prove that for all $n \varepsilon$ N, $(2n + 7) < (n + 3)^2$. **8**

Unit II

4. (a) Show that I (that set of all integers) is an abelian group w.r.t. addition. **8**

(b) Prove that group G is abelian iff $(ab)^2 = a^2b^2$ for all a, b ε G. **8**

5. (a) Prove that the set of all $n*n$ matrices form a ring with identity but it is not commutative ring with respect to matrix addition and multiplication. **8**

(b) Prove that the intersection of two subrings in a ring. **8**

Unit III

6. (a) If $B=\begin{bmatrix}2 & 3 & 0\\1 & -1 & 5\end{bmatrix}$, $C=\begin{bmatrix}1 & -2 & 3\\-1 & 0 & 2\end{bmatrix}$; find 3B + 4C. **8**

(b) Without using the concept of inverse of matrix, find the matrix $\begin{bmatrix}x & y\\z & u\end{bmatrix}$ such that $\begin{bmatrix}5 & -7\\-2 & 3\end{bmatrix}\begin{bmatrix}x & y\\z & u\end{bmatrix}=\begin{bmatrix}-16 & -6\\7 & 2\end{bmatrix}$. **8**

7. (a) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\3 & 2 & 1\\1 & 1 & 1\end{bmatrix}$. **8**

(b) Solve the following system of homogeneous equations :

$$3x + 2y + 7z = 0$$
$$4x - 3y - 2z = 0$$
$$5x + 9y + 23z = 0$$
8

Unit IV

8. (a) Find the characteristic roots and spectrum of matrix,

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ **8**

(b) If α is an eigen value of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is an eigen value of adj. A. **8**

9. State and prove Cayley-Hamilton theorem and verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. **16**