

BT-3/D-19

33080

## DISCRETE STRUCTURE

CSE-201N/IT-209N

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

## Unit I

1. (a) Let P, Q and R are three finite sets. Then prove that :

$$|P \cup Q \cup R| = |P| + |Q| + |R| - |P \cap Q| - |P \cap R| - |Q \cap R| + |P \cap Q \cap R|$$

Also draw Venn diagram.

8

- (b) Let A and B be two sets, then show that  $(A \cup B)^c = A^c \cap B^c$ . Also justify your answer by giving suitable example.

7

2. (a) Determine which propositions are the following by constructing Truth Tables :

(i)  $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

(ii)  $(p \leftrightarrow q) \rightarrow ((p \wedge q) \vee (\sim p \wedge q))$

8

- (b) Prove by Mathematical Induction for any integer  $n$ ,  $11^{n+2} + 12^{2n+1}$  is divisible by 133. 7

## Unit II

3. (a) Let  $A = \{a, b, c, d\}$ . Let  $R = \{(a, b), (a, c), (b, a), (b, c), (c, d), (d, a)\}$ . Find the Reflexive-transitive closure of  $R$ . 8
- (b) Find whether the relation  $(x, y) \in R$ , if  $x \geq y$  defined on the set of positive integers is a partial order relation or not. 7
4. Let  $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$  and let the relation  $R(\leq)$  be the relation (divides) a partial ordering on  $D_{100}$ .
- (a) Draw the Hasse Diagram for the above relation :
- (i) Determine the GLB of  $B$ , where  $B = \{10, 20\}$
  - (ii) Determine the LUB of  $B$ , where  $B = \{10, 20\}$
  - (iii) Determine the GLB of  $B$ , where  $B = \{5, 10, 20, 25\}$
  - (iv) Determine the LUB of  $B$ , where  $B = \{5, 10, 20, 25\}$ . 10
- (b) Determine whether  $(D_{100}, R)$  is a lattice or not. 5

### Unit III

5. (a) In a shipment, there are 40 floppy disks of which 5 are defective. Determine :

- (i) In how many ways can we select 5 floppy disks ?
- (ii) In how many ways can we select 5 non-defective floppy disks ?
- (iii) In how many ways can we select 5 floppy disks containing exactly 3 defective floppy disks ? 7

- (b) How many permutations can be made out of the letters of word "COMPUTER" ? How many of these :

- (i) Begin with C
- (ii) End with R
- (iii) Begin with C and end with R
- (iv) C and R occupy the end places. 8

6. (a) Solve the recurrence relation  $a_{r+2} - 2a_{r+1} + a_r = 2^r$  by the method of generating functions with the initial conditions  $a_0 = 2$  and  $a_1 = 1$ . 8

- (b) Find the particular solution of the difference equation  $a_{r+2} - 4a_r = r^2 + r + 1$ . 7

## Unit IV

7. (a) Consider an algebraic system  $(G, *)$ , where 'G' is the set of all non-zero real numbers and '\*' is a binary operation defined by  $a*b = ab/4$ . Show that  $(G, *)$  is an abelian group. 8
- (b) Let  $(I, +)$  be a group, where I is the set of all integers and '+' is an addition operation. Determine whether the following subsets of G are subgroups of G :
- (i) The set  $G_1$  of all odd integers
- (ii) The set  $G_2$  of all positive integers. 7
8. (a) Consider an algebraic system  $(Q, *)$ , where 'Q' is the set of all rational numbers and '\*' is a binary operation defined by  $a * b = a + b - ab$  for all  $a, b \in Q$ . Determine whether  $(Q, +)$  is a group or not. 8
- (b) Explain Ring Homomorphism with example. 7