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BT-4/J-21 **44151**

DISCRETE MATHEMATICS

Paper–PC-CS-202A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) Show that

$$1^2 + 3^2 + 5^2 + \dots\dots\dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

by mathematical induction.

(b) Given that

$(A \cap C) \subseteq (B \cap C)$

$(A \cap \bar{C}) \subseteq (B \cap \bar{C})$

show that $A \subseteq B$.

2. (a) Construct the truth tables for the following statements

(i) $(p \to p) \to (p \to \bar{p})$.

(ii) $(p \vee \bar{q}) \to \bar{p}$.

(iii) $p \leftrightarrow (\bar{p} \vee \bar{q})$.

(b) Let A, B and C be sets such that $(A \cap B \cap C) = \phi$, $(A \cap B) \neq \phi$, $(A \cap C) \neq \phi$ and $(B \cap C) \neq \phi$. Draw the corresponding Venn diagram.

UNIT–II

3. (a) Find all the partitions of B = $\{a, b, c, d\}$.

(b) Let A = $\{a, b\}$ and B = $\{4, 5, 6\}$. Given each of the following :

(i) $A \times B$

(ii) $B \times A$

(iii) $A \times A$

(iv) $B \times B$.

4. (a) Show that if R_1 and R_2 are equivalence relations on A, then $R_1 \cap R_2$ is an equivalence relation on A.

(b) Let A = Z, the set of integers and let R is defined by $a \mathrel{R} b$ if and only if $a \leq b$. Is R is an equivalence relation ?

UNIT–III

5. (a) Prove that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-to-one functions, then gof is one-to-one.

(b) Let A = B = C = R, and let $f : A \rightarrow B$ and $g : B \rightarrow C$ be defined by $f(a) = a - 1$ and $g(b) = b^2$ find :

(i) $(fog)\,(2)$

(ii) $(gof)\,(2)$

(iii) $(fof)\,(y)$

(iv) $(gog)\,(y)$.

6. (a) Let A and B be two finite set with same number of elements, and let $f : A \to B$ be an everywhere defined functions :

(i) If f is one-to-one, then f is onto.

(ii) If f is onto, then f is one-to-one.

(b) If n pigeons are assigned to m pigeonholes, and $m < n$, then atleast one pigeonhole contains two or more pigeons.

UNIT–IV

7. (a) Define the following :

(i) Group.

(ii) Cyclic group.

(b) Let H and K be subgroups of group G :

(i) Prove that $H \cap K$ is subgroup of G.

(ii) Show that $H \cup K$ need not be subgroup of G.

8. (a) Let G be an Abelian group and n is a fixed integer. Prove that the function $f : G \to G$ defined by $f(a) = a^n$, for $a \in G$ is a homomorphism.

(b) Let G be a group, and let $H = \{x/x \in G \text{ and } ax = xa \text{ for all } a \in G\}$. Show that H is a normal subgroup of G.