Subject Code: KAS103

PAPER ID-411938

Roll No: 

**BTECH**
**(SEM I) THEORY EXAMINATION 2021-22**
**MATHEMATICS-I**

***Time: 3 Hours*** ***Total Marks: 100***

**Notes:**

- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

| **SECTION-A** | Attempt **All** of the following Questions in brief Marks**(10X2=20)** | |
|---|---|---|
| Q1(a) | Find the eigen value of $A^3$ where A=$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. | 1 |
| Q1(b) | Show that the system of vectors $X_1 = (1, -1, 1), X_2 = (2, 1, 1), and$ $X_3 = (3, 0, 2)$ are linearly dependent or linearly independent. | 1 |
| Q1(c) | If $y = A\ sin\ nx + B \cos nx$ , prove that $y_2 + n^2 y = 0$. | 2 |
| Q1(d) | Find the asymptotes parallel to y-axis of the curve $\frac{a^2}{x} + \frac{b^2}{y} = 1$. | 2 |
| Q1(e) | If $x = rcos\theta , y = rsin\theta, find$ $\frac{\partial(r,\theta)}{\partial(x,y)}$. | 3 |
| Q1(f) | An error of 2% is made in measuring length and breadth then find the percentage error in the area of the rectangle. | 3 |
| Q1(g) | Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\ dy\ dx$. | 4 |
| Q1(h) | Find the volume common to the cylinders $x^2 + y^2 = a^2\ and\ x^2 + z^2 = a^2$. | 4 |
| Q1(i) | Find p such that $\vec{A} = (px + 4y^2 z)i + (x^3 sinz - 3y)j - (e^x + 4 \cos x^2 y)k$ is solenoidal. | 5 |
| Q1(j) | State Green's theorem for a plane region. | 5 |

| **SECTION-B** | Attempt **ANY THREE** of the following Questions Marks**(3X10=30)** | |
|---|---|---|
| Q2(a) | Find the eigen values and corresponding eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. | 1 |
| Q2(b) | Verify Rolle's theorem for the function $f(x) = \sqrt{4 - x^2}$ $in\ [-2, 2]$. | 2 |
| Q2(c) | Find the first six terms of the expansions of the function $e^x \log(1 + y)$ in a Taylor series in the neighborhood of the point $(0, 0)$. | 3 |
| Q2(d) | Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xydydx$ and hence evaluate the same. | 4 |
| Q2(e) | If a vector field is given by $\vec{F} = (x^2 - y^2 + x)i - (2xy + y)j$ Is this field irrotational? If so, find its scalar potential. | 5 |

| **SECTION-C** | Attempt **ANY ONE** following Question Marks **(1X10=10)** | |
|---|---|---|
| Q3(a) | Find for what values of $\lambda\ and\ \mu$ the system of linear inequation: $x + y + z = 6$, $x + 2y + 5z = 10$ ,$2x + 3y + \lambda z = \mu$ has (i) a unique solution, (ii) no solution, (iii) infinite solution. Also find the solution for $\lambda = 2\ and\ \mu = 8$. | 1 |
| Q3(b) | Find the rank of matrix reducing it to normal form $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$ | 1 |

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| **SECTION-C** | Attempt **ANY ONE** following Question | Marks **(1X10=10)** |
|---|---|---|
| Q4(a) | If $y = (sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ and calculate $y_n(0)$. | 2 |
| Q4(b) | Verify mean value theorem for the function $f(x) = x(x-1)(x-2)\ in\ \left[0, \frac{1}{2}\right]$. | 2 |

| **SECTION-C** | Attempt **ANY ONE** following Question | Marks **(1X10=10)** |
|---|---|---|
| Q5(a) | A rectangular box which is open at the top having capacity 32c.c.Find the dimension of the box such that the least material is required for its constructions. | 3 |
| Q5(b) | If u, v and w are the roots of $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ , cubic in $\lambda$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. | 3 |

| **SECTION-C** | Attempt **ANY ONE** following Question | Marks **(1X10=10)** |
|---|---|---|
| Q6(a) | Find by double integration the area enclosed by the pair of curves $y = 2 - x \quad and \quad y^2 = 2(2-x)$ | 4 |
| Q6(b) | Find C.G. of the area in the positive quadrant of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ . | 4 |

| **SECTION-C** | Attempt **ANY ONE** following Question | Marks **(1X10=10)** |
|---|---|---|
| Q7(a) | Find the directional derivative of $f(x,y,z) = xyz$ at the point $P(1,-1,2)$ in the direction of the vector $(2i - 2j + 2k)$. | 5 |
| Q7(b) | Verify Stoke's Theorem for $\vec{F} = (y - z + 2)i + (yz + 4)j - (xz)k$ over the surface of cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the XOY plane. | 5 |