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# B. TECH. <br> THEORY EXAMINATION (SEM-IV) 2016-17 <br> ENGINEERING MATHEMATICS-III 

Time: 3 Hours
Max. Marks : 100

## SECTION - A

1. Attempt all parts of the following question:
$(2 \times 10=20)$
(a) Evaluate $\int_{C} \frac{e^{z}}{z+1} d z$, where $C$ is the circle $|z|=2$
(b) Prove that $f(z)=\sinh z$ is analytic
(c) Prove that Modulation theorem $F\{\mathrm{f}(x) \cos a x\}=\frac{1}{2}[f(\mathrm{~s}+a)+f(\mathrm{~s}-a)]$
(d) Solve the Z-transform: $y_{k+2}+y_{k+1}-2 y_{k}=0, y_{0}=4 y_{1}=0$
(e) What is the meaning of Skewness?
(f) Write Normal equation of $y=a+\frac{b}{x}$
(g) Prove that $\Delta+\nabla=\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$
(h) Find first approximation value of (17) $)^{1 / 3}$ by using Newton Raphson method
(i) Using Picard's method , find the solution of $\frac{d y}{d x}=1+x y$ upto the third approximation when $x(0)=0$
(j) Find $\mathrm{y}(0.1)$ using Euler's method given that $\frac{d y}{d x}=\log (x+y) \quad y(0)=1.0$

## SECTION - B

2. Attempt any five parts of the following question:
$(5 \times 10=50)$
(a) Prove that the function $f(z)$ defined by $f(z)=\left\{\begin{array}{cl}\frac{x^{3}(1+i)-y^{3}(1-i)}{x^{2}+y^{2}} & z \neq 0 \\ 0 & z=0\end{array}\right.$ is continuous and the C.R. equations are satisfied at the origin, yet $f^{\prime}(0)$ does not exist.
(b) Using Cauchy Integral formula to evaluate $\int_{C} \frac{e^{2 z}}{(z+1)^{4}} d z$, where C is the circle $|z|=3$.
(c) Find the Fourier cosine transform of $\frac{1}{1+x^{2}}$ and then find Fourier sine transform of $\frac{x}{1+x^{2}}$.
(d) Find the multiple linear regression of $X_{1}$ on $X_{2}$ and $X_{3}$ from the data relating to three variables:

| $\mathrm{X}_{1}$ | 7 | 12 | 17 | 20 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{2}$ | 4 | 7 | 9 | 12 |
| $\mathrm{X}_{3}$ | 1 | 2 | 5 | 8 |

(e) Find the root of the equation $x e^{x}=\cos x$ by using Regula-Falsi method correct to four decimal places.

$$
2 x+3 y+z=9
$$

(f) Apply Crout's method and solve the system of equations $x+2 y+3 z=6$

$$
3 x+y+2 z=8
$$

(g) Find the value $\mathrm{y}(1.1)$ using Runge-Kutta method of fourth order, given that $\frac{d y}{d x}=y^{2}+x y, y(1)=1.0$, take $\quad h=0.05$

## SECTION - C

Attempt any two questions of the following:
( $\mathbf{2} \times 15=30$ )
3. (i) Show that the function defined by $f(z)=\sqrt{|x y|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied
(ii) Evaluate: $\int_{0}^{2 \pi} \frac{d \theta}{a+b \sin \theta}$ if $a>|b|$
(iii) Solve by Z-transform: $y_{k+2}-4 y_{k+1}+3 y_{k}=5^{k}$
4. (i) Using the convolution theorem, evaluate $Z^{-1}\left\{\frac{z^{2}}{(z-1)(z-3)}\right\}$
(ii) If the $\theta$ is the acute angle between the two regression lines in the case of two variables $x$ and $y$, show that $\tan \theta=\frac{1-r^{2}}{r} \cdot \frac{\sigma_{x} \sigma_{y}}{\sigma_{x}{ }^{2}+\sigma_{y}{ }^{2}}$, where $r, \sigma_{x}, \sigma_{y}$ have their usual meanings. Explain the significance of the formula when $r=0$ and $r= \pm 1$
(iii) By using $\chi^{2}$-test, find out whether there is any association between income level and type of schooling:

| Social status Health | Poor | Rich | Total |
| :--- | :---: | :---: | :---: |
| Below Normal | 130 | 20 | 150 |
| Normal | 102 | 108 | 210 |
| Above Normal | 24 | 96 | 120 |
| Total | 256 | 224 | 480 |

5. (i) Find the missing figure in the following table

| $x$ | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 45 | 49.2 | 54.1 | $?$ | 67.4 |

(ii) Find a cubic polynomial which approximates the data:

| $x$ | -2 | -1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| $y(x)$ | -12 | -8 | 3 | 5 |

(iii) Find an approximate value of the $\log _{e} 5$ by calculating to four decimal places by Simpson's $\frac{1}{3}$ rule, given $\int_{0}^{5} \frac{d x}{4 x+5}$

